Vyšetřete průběhy funkcí a načrtněte jejich grafy. Prozkoumejte také chování v nekonečnu a v okolí bodů nespojitosti.

Pozmámka: Derivace jsou vypočteny pomocí programového balíku Maple a už jsem je ručně neopravoval. Proto je použit následující zápis: $\ln(x)^2$ znamená $\ln^2 x$, tj. $(\ln x)^2$. Nespleťte si to s $\ln(x^2)$!

1. $f(x) = 3x - x^3$
2. $f(x) = x^4 - x^2$
3. $f(x) = (x-1)^3(x+1)^3$
4. $f(x) = x^3 - 6x^2 + 9x - 4$
5. $f(x) = x^2(x-6)$
6. $f(x) = x^3 - 2x^2 + x$
7. $f(x) = (x+2)(x^2-1)$
8. $f(x) = x^5 - 5x + 1$
9. $f(x) = x^3 - 6x^2 + 9x - 4$
10. $f(x) = (x-1)^2(x+1)$
11. $f(x) = x^2(x^2-1)^3$
12. $f(x) = \sqrt{x+3}x^2$
13. $f(x) = \dfrac{x^2-1}{x+2}$
14. $f(x) = \dfrac{x^3-3x^2+3x+1}{x-1}$
15. $f(x) = \dfrac{x^2-1}{x^2+2}$
16. $f(x) = \dfrac{x^2-1}{x}$
17. $f(x) = \dfrac{x^2+x+1}{(x+1)^2}$
18. $f(x) = \dfrac{x}{(x-1)^2}$
19. $f(x) = \dfrac{x^2+1}{x^2-1}$
20. $f(x) = \dfrac{x}{(x+1)^2}$
21. $f(x) = \dfrac{2x-1}{(x-1)^2}$
22. $f(x) = \dfrac{x^3}{x-1}$
23. $f(x) = 4\dfrac{1}{x} + \dfrac{1}{x^4}$
24. $f(x) = \dfrac{x^2}{x^2-4}$
25. $f(x) = \dfrac{1}{2}\dfrac{x^3+2}{x}$
26. $f(x) = \dfrac{(x-1)^2}{x^2+1}$
27. $f(x) = \dfrac{1-x^3}{x^2}$
28. $f(x) = 3\dfrac{1}{x} - \dfrac{1}{x^3}$
29. $f(x) = 2\dfrac{1}{x} - \dfrac{1}{x^2}$
30. $f(x) = \dfrac{1}{(x-1)(x-2)(x-3)}$
31. $f(x) = \dfrac{(x+1)^2}{(1-x)^2}$
32. $f(x) = \dfrac{(x+1)^4}{(1-x)^4}$
33. $f(x) = \dfrac{4-3x}{x-x^2}$
34. $f(x) = \dfrac{x-2}{x^2}$
35. $f(x) = \dfrac{x^3}{3-x^2}$
36. $f(x) = \dfrac{x^2+1}{x}$
37. $f(x) = \dfrac{x}{\ln(x)}$
38. $f(x) = \dfrac{1}{2}\dfrac{x^3}{(x+1)^2}$
39. $f(x) = \ln(x)\sqrt{x}$
40. $f(x) = \dfrac{\ln(x)^2}{x}$
41. $f(x) = x\ln(x)^2$
42. $f(x) = xe^x$
43. $f(x) = x + \ln(x^2)$
44. $f(x) = xe^{(-x)}$
45. $f(x) = (x+1)e^{(-x^2)}$
46. $f(x) = (x^2+x+1)e^x$
47. $f(x) = e^{(-x)} - e^{(-2x)}$
48. $f(x) = \dfrac{e^x}{x+1}$
49. $f(x) = x^2e^{(-x)}$
50. $f(x) = e^{(\frac{x-1}{x^2})}$
51. $f(x) = \sqrt{1-e^{(-x^2)}}$
52. $f(x) = \arccos\dfrac{1-x}{1-2x}$
53. $f(x) = \arctan\dfrac{1}{x}$
54. $f(x) = e^{\frac{x-1}{x^3}}$
55. $f(x) = x^2e^{\frac{1}{x}}$

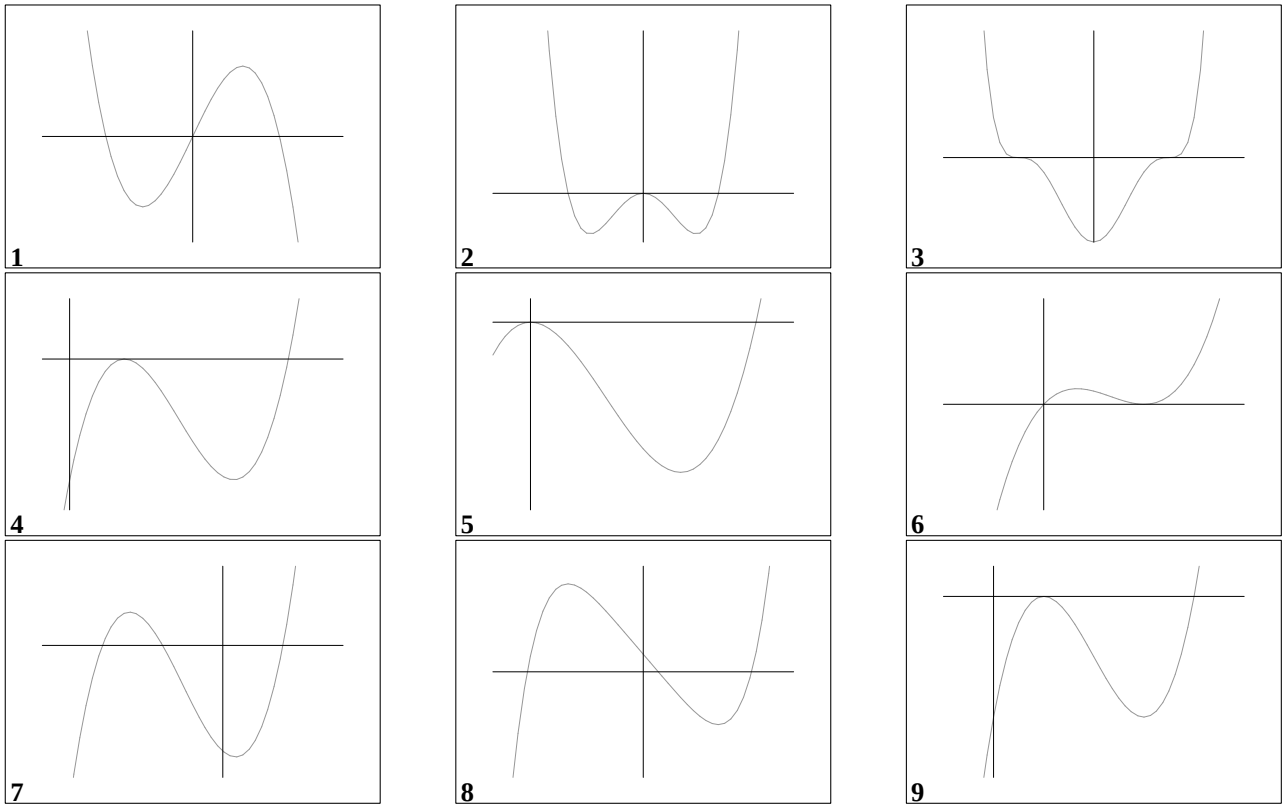
Derivatives: 1. $y' = -3(x-1)(x+1), y'' = -6x$. **2.** $y' = 2x(2x^2-1), y'' = 12x^2-2$. **3.** $y' = 6x(x-1)^2(x+1)^2, y'' = 6(x-1)(x+1)(5x^2-1)$. **4.** $y' = 3(x-1)(x-3), y'' = 6x-12$. **5.** $y' = 3x(x-4), y'' = 6x-12$. **6.** $y' = (3x-1)(x-1), y'' = 6x-4$. **7.** $y' = 3x^2-1+4x, y'' = 6x+4$. **8.** $y' = 5(x-1)(x+1)(x^2+1), y'' = 20x^3$. **9.** $y' = 3(x-1)(x-3), y'' = 6x-12$. **10.** $y' = (3x+1)(x-1), y'' = 6x-2$. **11.** $y' = 2x(2x-1)(2x+1)(x-1)^2(x+1)^2, y'' = 2(x-1)(x+1)(28x^4-17x^2+1)$. **12.** $y' = \dfrac{1}{2}\dfrac{x(5x+12)}{\sqrt{x+3}}, y'' = \dfrac{3}{4}\dfrac{5x^2+24x+24}{(x+3)^{(3/2)}}$.

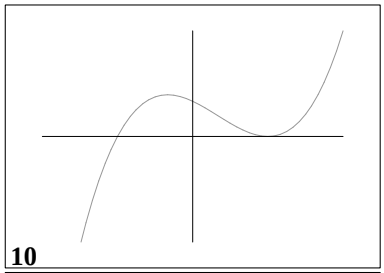
13. $y' = \dfrac{x^2+4x+1}{(x+2)^2}, y'' = 6\dfrac{1}{(x+2)^3}$. **14.** $y' = 2\dfrac{(x-2)(x^2-x+1)}{(x-1)^2}, y'' = 2\dfrac{x^3-3x^2+3x+1}{(x-1)^3}$. **15.** $y' = 6\dfrac{x}{(x^2+2)^2}, y'' = -6\dfrac{3x^2-2}{(x^2+2)^3}$. **16.** $y' = \dfrac{x^2+1}{x^2}, y'' = -2\dfrac{1}{x^3}$. **17.** $y' = \dfrac{x-1}{(x+1)^3}, y'' = -2\dfrac{x-2}{(x+1)^4}$.

18. $y' = -\dfrac{x+1}{(x-1)^3}, y'' = 2\dfrac{x+2}{(x-1)^4}$. **19.** $y' = -4\dfrac{x}{(x-1)^2(x+1)^2}, y'' = 4\dfrac{3x^2+1}{(x-1)^3(x+1)^3}$. **20.** $y' = -\dfrac{x-1}{(x+1)^3}, y'' = 2\dfrac{x-2}{(x+1)^4}$. **21.** $y' = -2\dfrac{x}{(x-1)^3}, y'' = 2\dfrac{2x+1}{(x-1)^4}$. **22.** $y' = \dfrac{x^2(2x-3)}{(x-1)^2}, y'' = 2\dfrac{x(x^2-3x+3)}{(x-1)^3}$. **23.** $y' = -4\dfrac{(x+1)(x^2-x+1)}{x^5}, y'' = 4\dfrac{2x^3+5}{x^6}$. **24.** $y' = -8\dfrac{x}{(x-2)^2(x+2)^2}, y'' = 8\dfrac{3x^2+4}{(x-2)^3(x+2)^3}$. **25.** $y' = \dfrac{(x-1)(x^2+x+1)}{x^2}, y'' = \dfrac{x^3+2}{x^3}$. **26.** $y' = 2\dfrac{(x-1)(x+1)}{(x^2+1)^2}, y'' = -4\dfrac{x(x^2-3)}{(x^2+1)^3}$. **27.** $y' = -\dfrac{x^3+2}{x^3}, y'' = 6\dfrac{1}{x^4}$. **28.** $y' = -3\dfrac{(x-1)(x+1)}{x^4}, y'' = 6\dfrac{x^2-2}{x^5}$. **29.** $y' = -2\dfrac{x-1}{x^3}, y'' = 2\dfrac{2x-3}{x^4}$. **30.** $y' = -\dfrac{3x^2-12x+11}{(x-1)^2(x-2)^2(x-3)^2}, y'' = \ldots$. **31.** $y' = -4\dfrac{x+1}{(x-1)^3}, y'' = 8\dfrac{x+2}{(x-1)^4}$. **32.**

$y' = -8\frac{(x+1)^3}{(x-1)^5}, y'' = 16\frac{(x+1)^2(x+4)}{(x-1)^6}$. **33.** $y' = -\frac{(3x-2)(x-2)}{x^2(x-1)^2}, y'' = 2\frac{-12x^2+12x+3x^3-4}{x^3(x-1)^3}$. **34.** $y' = -\frac{x-4}{x^3}, y'' = 2\frac{x-6}{x^4}$. **35.** $y' = -\frac{x^2(x-3)(x+3)}{(-3+x^2)^2}, y'' = -6\frac{x(9+x^2)}{(-3+x^2)^3}$. **36.** $y' = \frac{(x-1)(x+1)}{x^2}$, $y'' = 2\frac{1}{x^3}$. **37.** $y' = \frac{\ln(x)-1}{\ln(x)^2}, y'' = -\frac{\ln(x)-2}{\ln(x)^3 x}$. **38.** $y' = \frac{1}{2}\frac{x^2(x+3)}{(x+1)^3}, y'' = 3\frac{x}{(x+1)^4}$. **39.** $y' = \frac{1}{2}\frac{2+\ln(x)}{\sqrt{x}}, y'' = -\frac{1}{4}\frac{\ln(x)}{x^{(3/2)}}$. **40.** $y' = -\frac{\ln(x)(-2+\ln(x))}{x^2}, y'' = 2\frac{1-3\ln(x)+\ln(x)^2}{x^3}$. **41.** $y' = \ln(x)(\ln(x)+2), y'' = 2\frac{\ln(x)+1}{x}$. **42.** $y' = e^x(x+1), y'' = e^x(x+2)$. **43.** $y' = \frac{x+2}{x}, y'' = -2\frac{1}{x^2}$. **44.** $y' = -e^{(-x)}(x-1), y'' = e^{(-x)}(x-2)$. **45.** $y' = -e^{(-x^2)}(-1+2x^2+2x), y'' = 2e^{(-x^2)}(x-1)(2x^2+4x+1)$. **46.** $y' = e^x(x+2)(x+1), y'' = e^x(5+5x+x^2)$. **47.** $y' = -e^{(-x)} + 2e^{(-2x)}, y'' = e^{(-x)} - 4e^{(-2x)}$. **48.** $y' = \frac{e^x x}{(x+1)^2}, y'' = \frac{e^x(x^2+1)}{(x+1)^3}$. **49.** $y' = -xe^{(-x)}(x-2), y'' = e^{(-x)}(2-4x+x^2)$. **50.** $y' = -\frac{(x-2)e^{(\frac{x-1}{x^2})}}{x^3}$, $y'' = \frac{e^{(\frac{x-1}{x^2})}(2x^3-5x^2-4x+4)}{x^6}$. **51.** $y' = \frac{xe^{(-x^2)}}{\sqrt{1-e^{(-x^2)}}}, y'' = \ldots$. **52.** $y' = -\frac{1}{(-1+2x)^2\sqrt{\frac{x(-2+3x)}{(-1+2x)^2}}}$, $y'' = \frac{12x^2-9x+1}{(-1+2x)^5(\frac{x(-2+3x)}{(-1+2x)^2})^{(3/2)}}$. **53.** $y' = -\frac{1}{x^2+1}, y'' = 2\frac{x}{(x^2+1)^2}$. **54.** $y' = -\frac{(2x-3)e^{(\frac{x-1}{x^3})}}{x^4}$, $y'' = \frac{e^{(\frac{x-1}{x^3})}(6x^4-12x^3+4x^2-12x+9)}{x^8}$. **55.** $y' = e^{(\frac{1}{x})}(-1+2x), y'' = \frac{e^{(\frac{1}{x})}(2x^2-2x+1)}{x^2}$.

Pictures:

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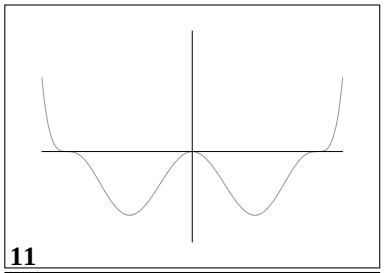
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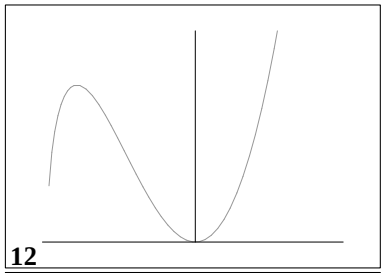
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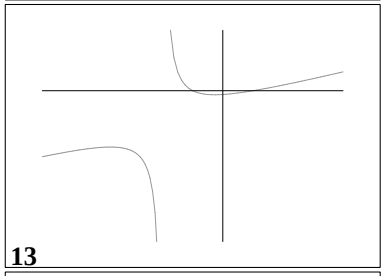
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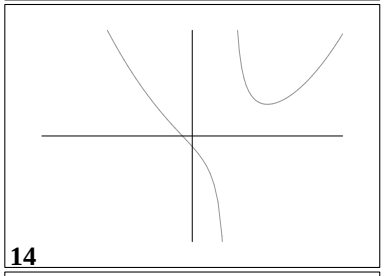
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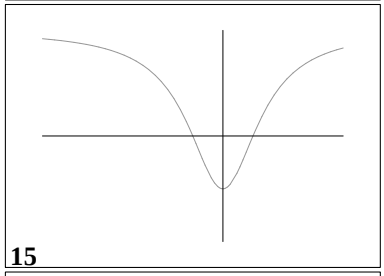
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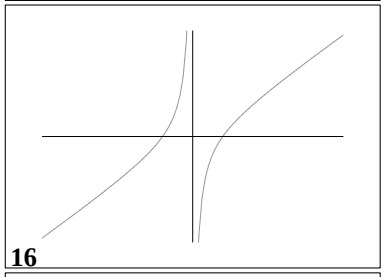
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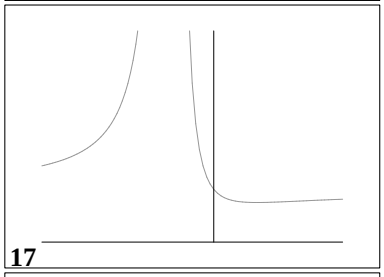
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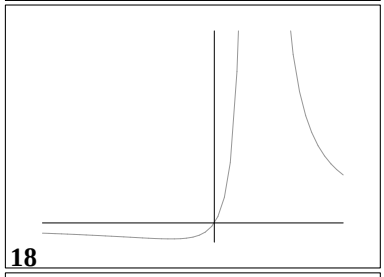
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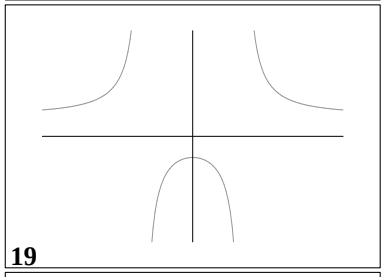
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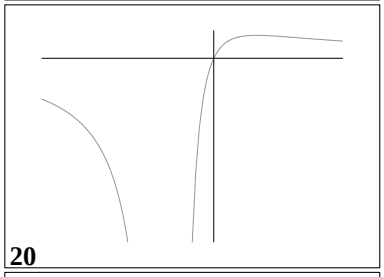
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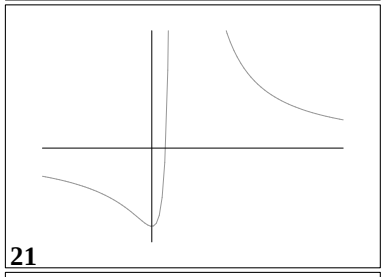
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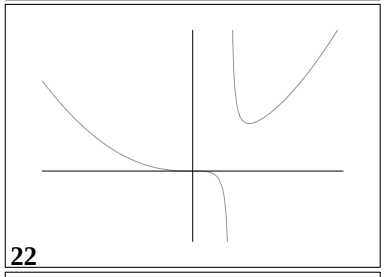
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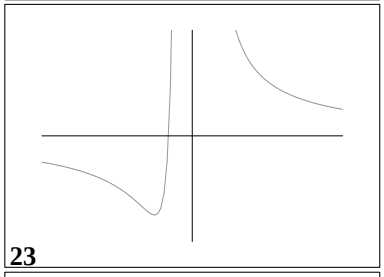
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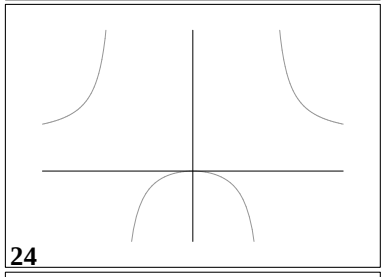
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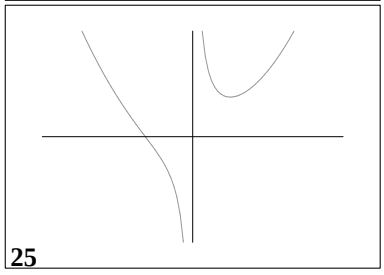
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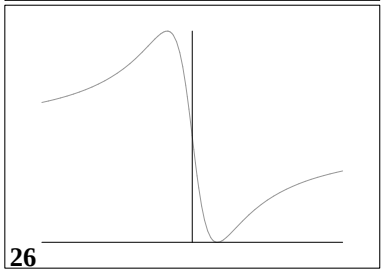
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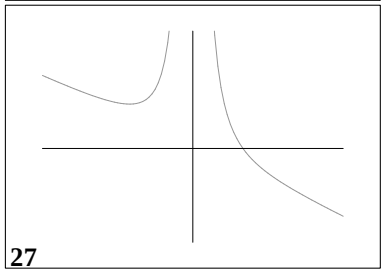
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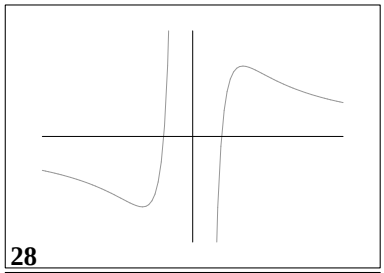
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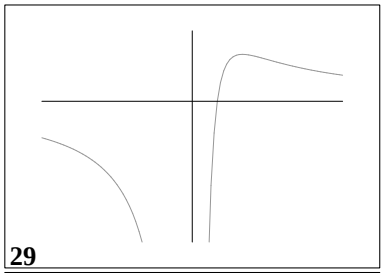
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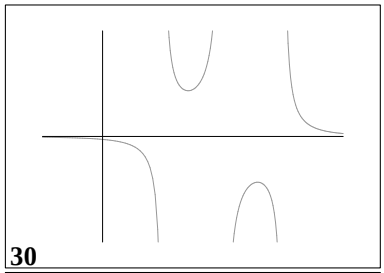
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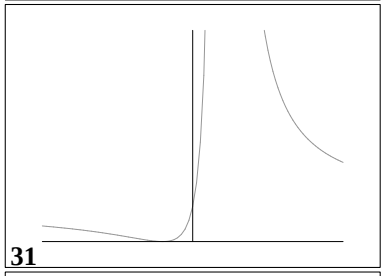
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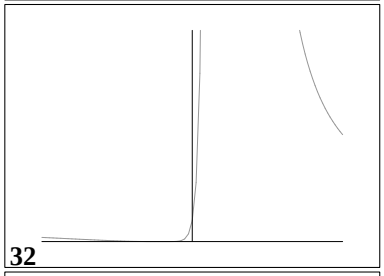
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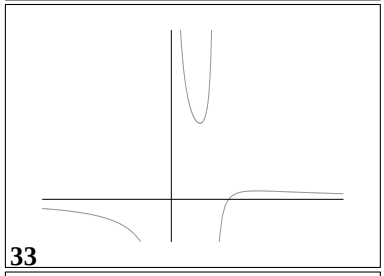
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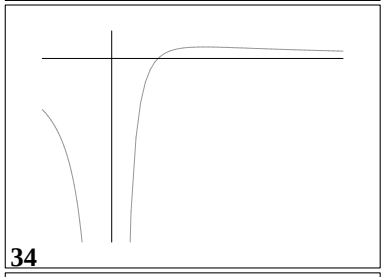
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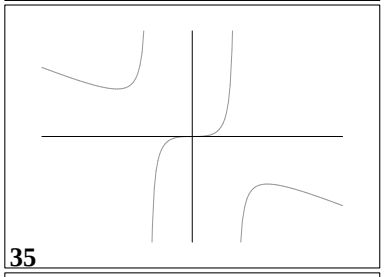
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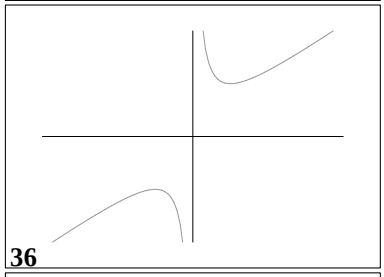
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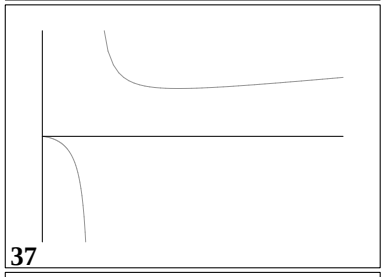
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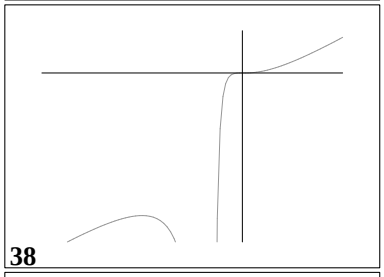
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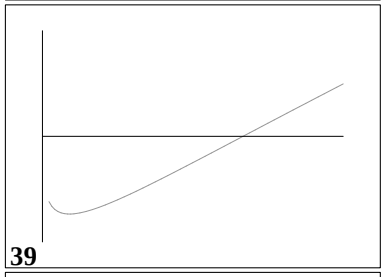
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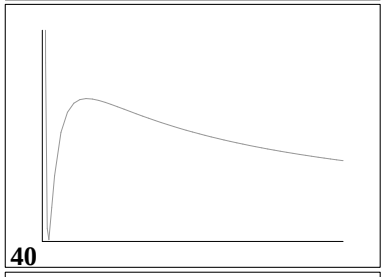
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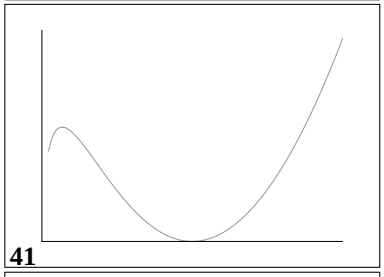
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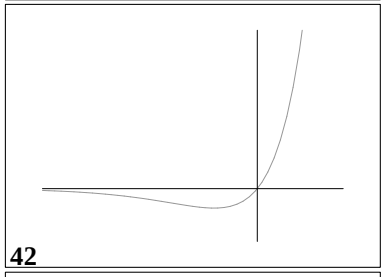
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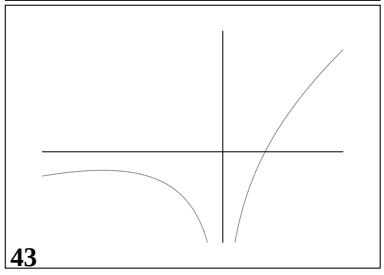
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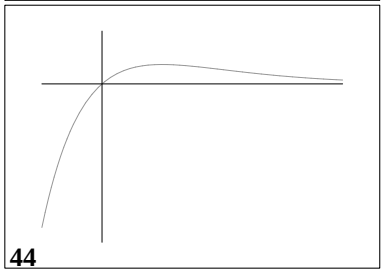
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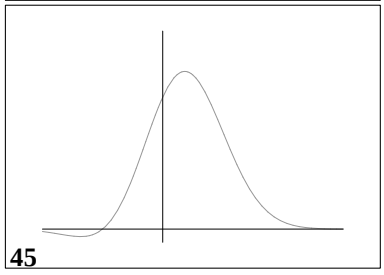
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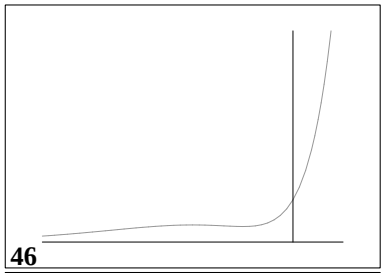
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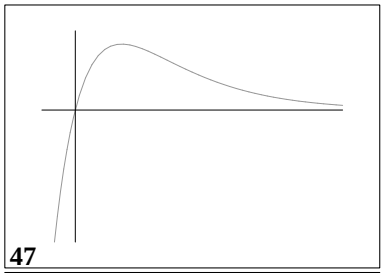
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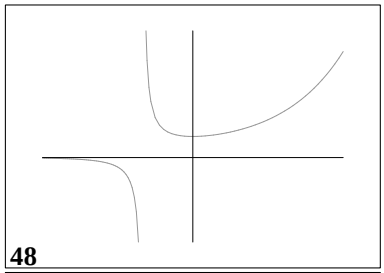
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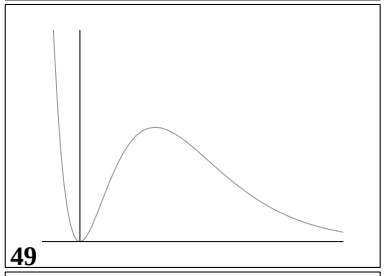
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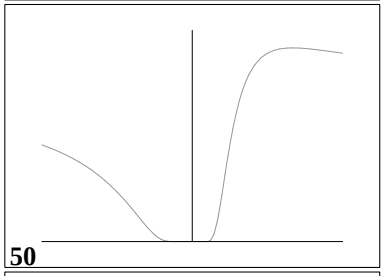
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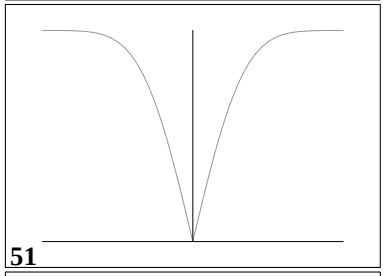
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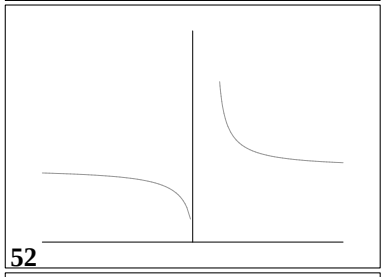
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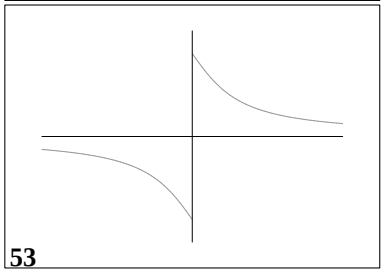
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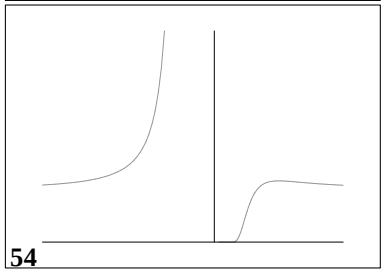
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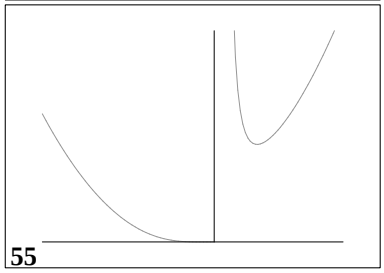
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